Wanderings of an Astronomer

Lesson 08

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Some of today's words & expressions :

- length contraction 时间扩张
- events
- simultaneity 同时性
- cause and effect 因果

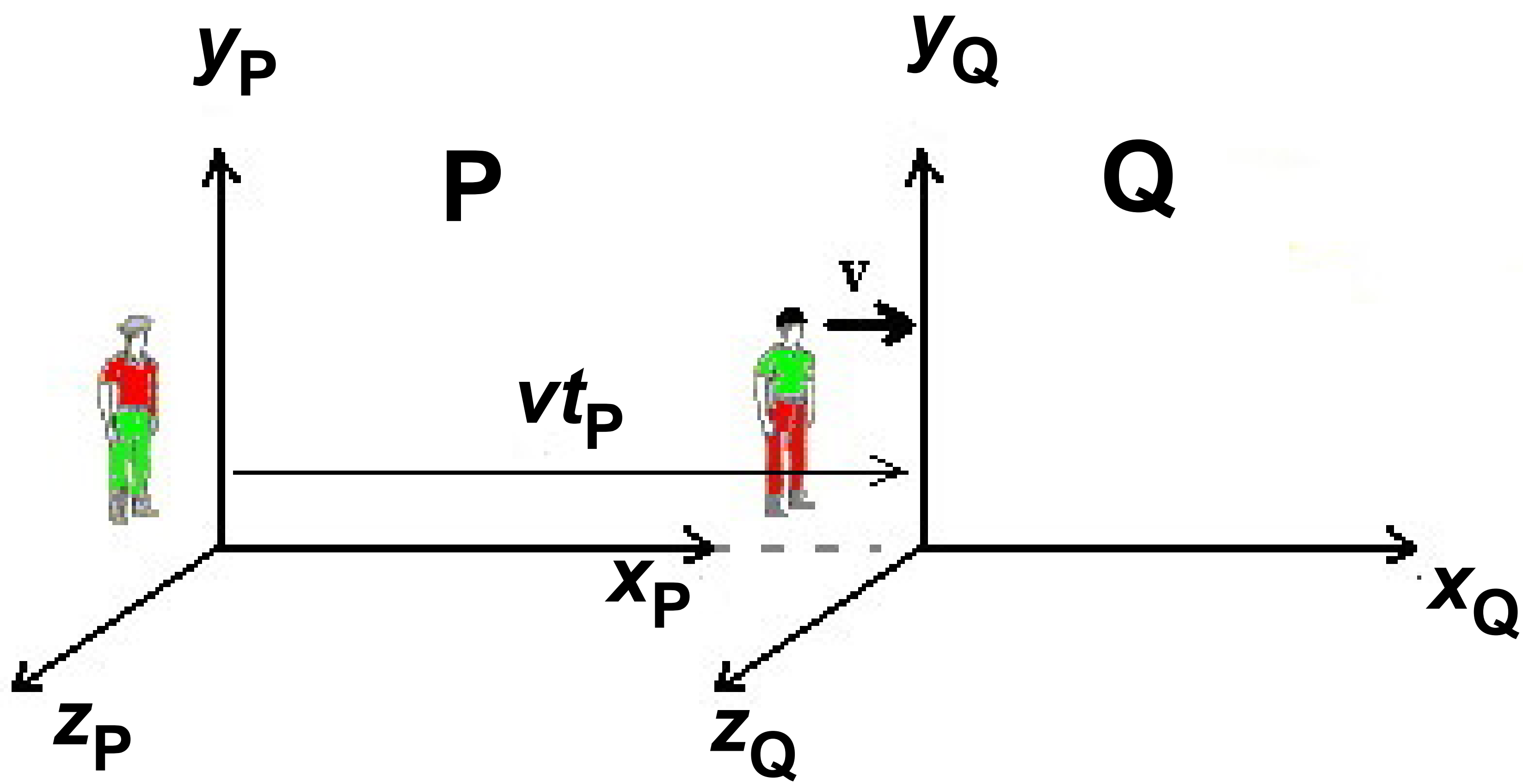
Special relativity 狭义相对论 (continued)

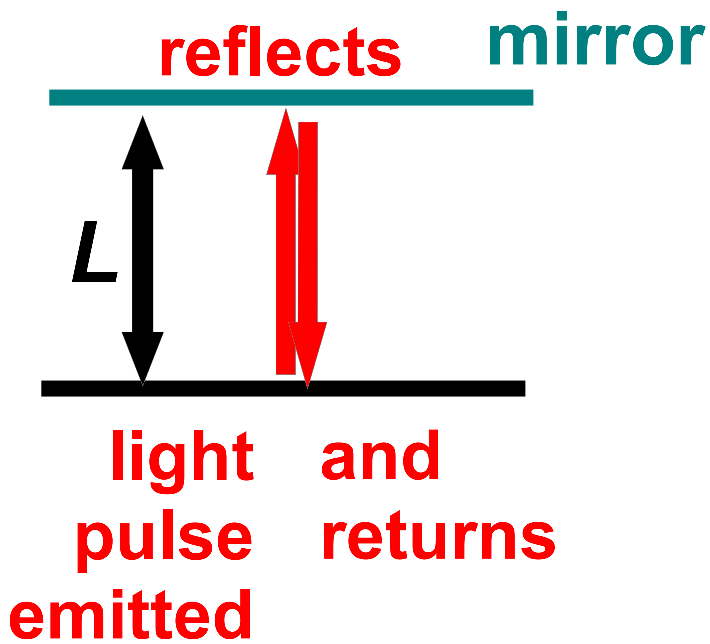
Imagine two inertial reference frames P & Q

Q is moving along x_P axis at speed v.

At time $t_P = t_Q = 0$, $x_P = x_Q = 0$

In Newtonian physics, at time t, $x_P = x_Q + vt$





Imagine a ‘clock’
in which a pulse
of light moves
in the +*y* direction,
then at distance L
reflects back
to start point.

tick
tock
tick
tock
...

Time for a ‘tick’ 滴答

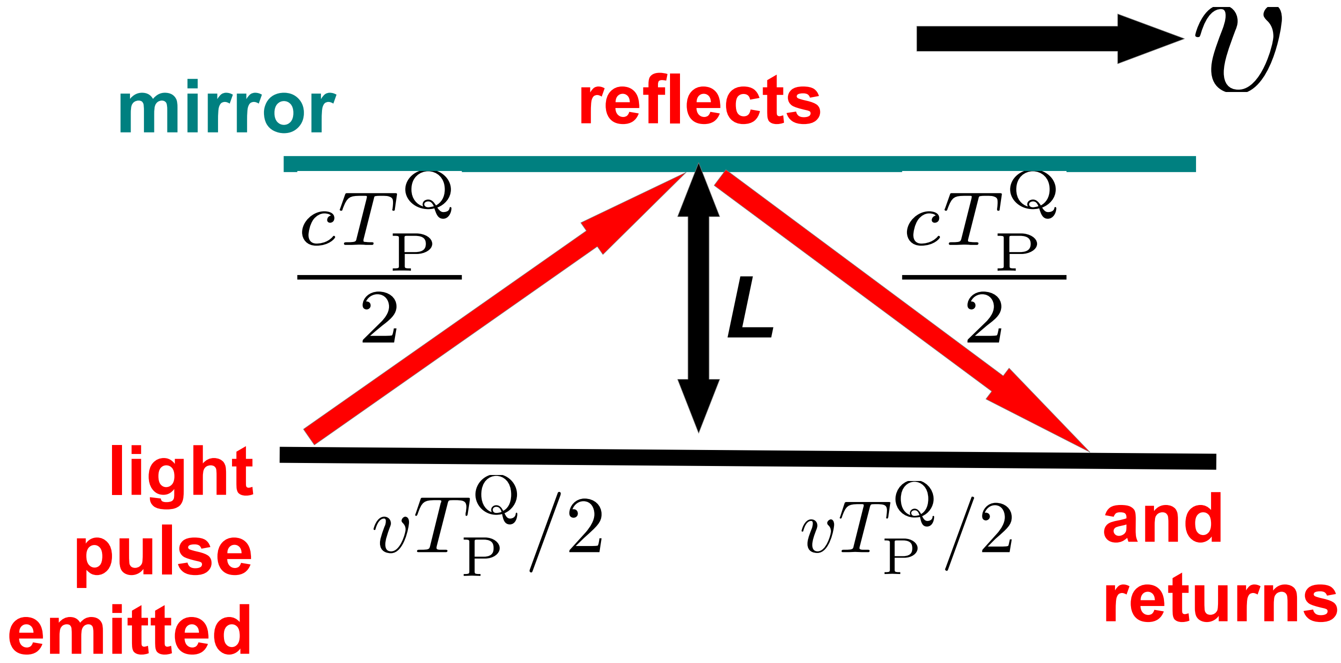
measured in P for a ‘clock’ at rest in P

or measured in Q for a ‘clock’ at rest in Q

clock

$$T_{\mathrm{P}}^{\mathrm{P}} = T_{\mathrm{Q}}^{\mathrm{Q}} = \frac{2L}{c}$$

frame

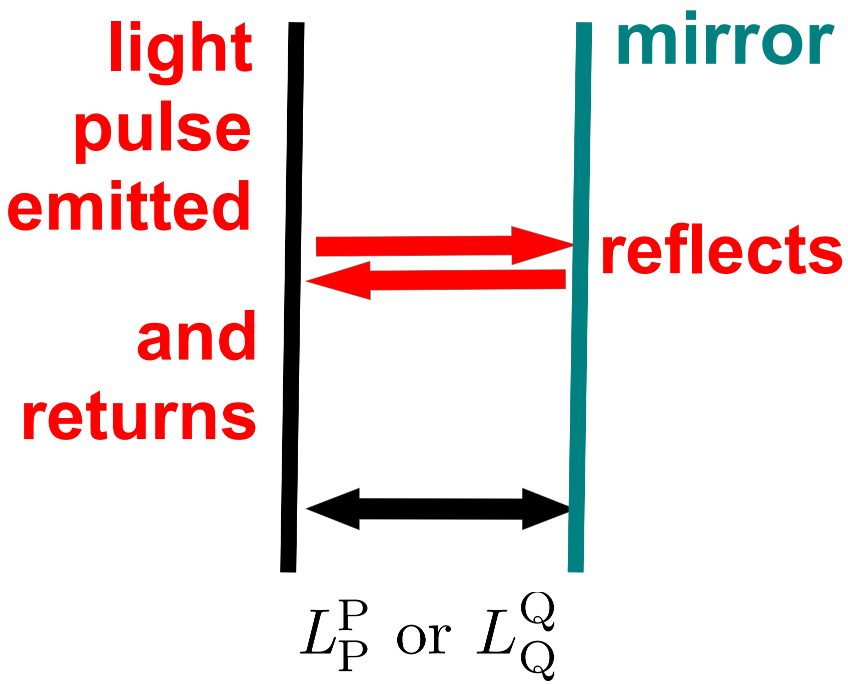


Now imagine 'clock' in Q timed from P – Tick takes time T_P^Q

or $\quad T_P^Q = \gamma T_P^P \quad$ where $\quad \gamma = \frac{1}{\sqrt{(1-v^2/c^2)}} \geq 1$

Observer in P sees moving clock (in Q) run slow by factor γ (Greek letter gamma)
'time dilation' 时间扩张
… and an observer in Q sees the (moving) clock in P run slow by the same factor.
Both are equally valid descriptions of nature.
Time is not absolute but *relative* !

Now turn ‘clock’ through 90º



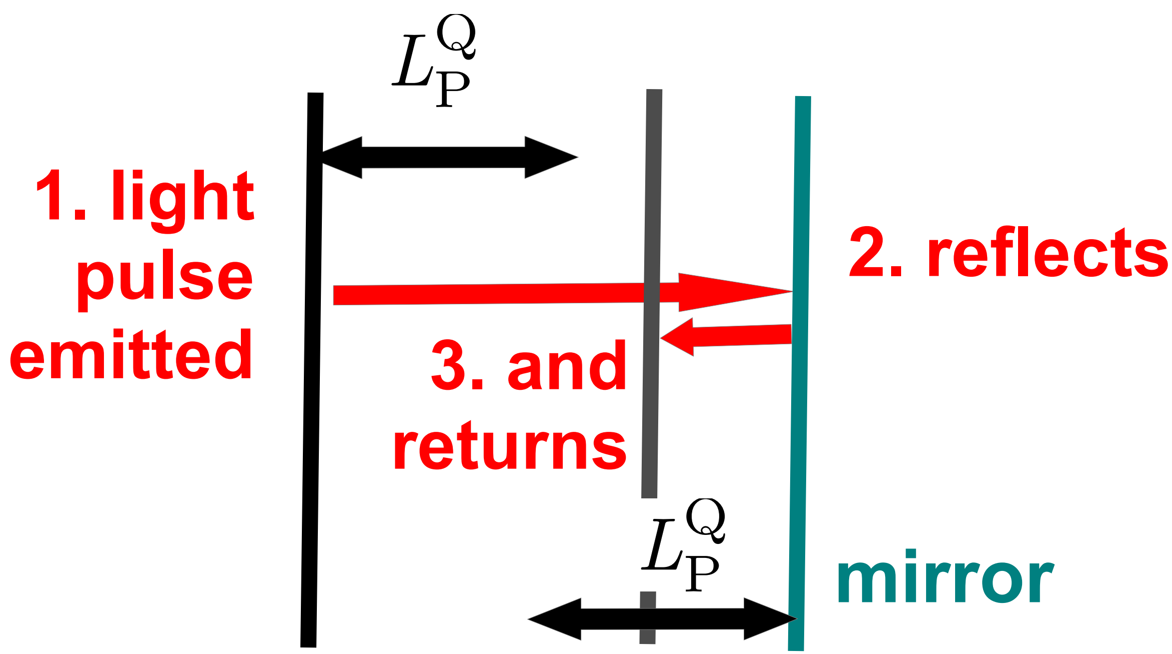
clock

$$T_{\mathrm{P}}^{\mathrm{P}} = 2L_{\mathrm{P}}^{\mathrm{P}}/c \qquad T_{\mathrm{Q}}^{\mathrm{Q}} = 2L_{\mathrm{Q}}^{\mathrm{Q}}/c$$

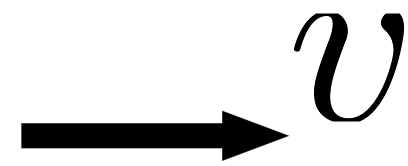
frame

same as before

Consider 'clock' in Q timed from P

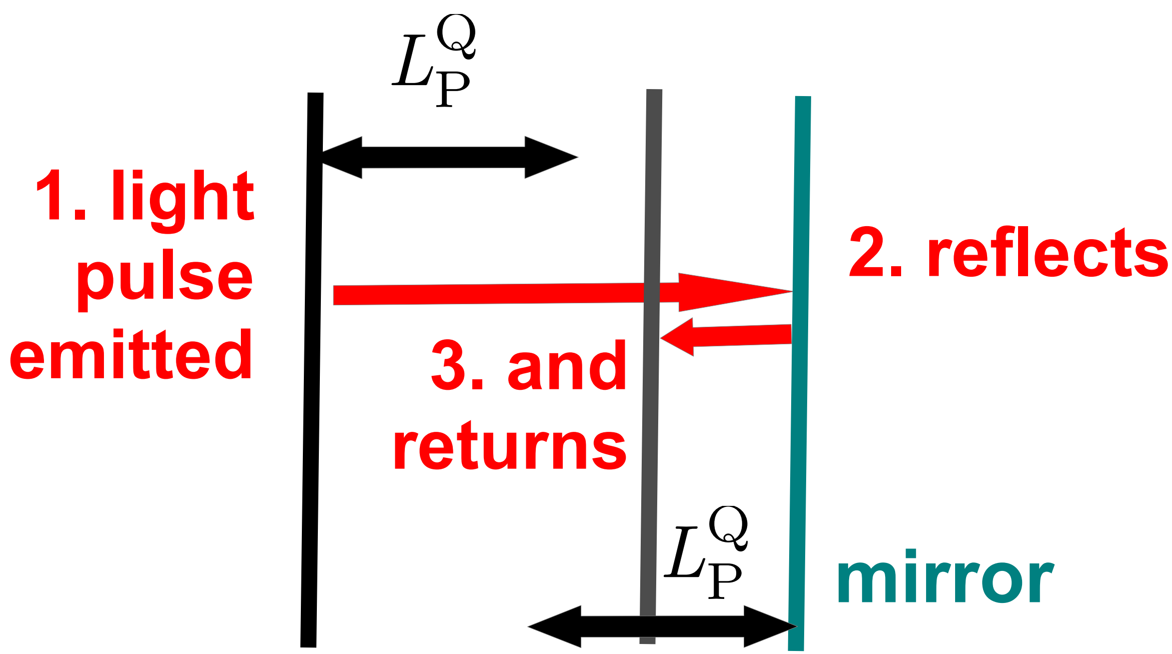


$$\text{Time } 1,2 \;= t_{\mathrm{P}}^{\mathrm{Q},1,2}$$
$$\text{Time } 2,3 \;= t_{\mathrm{P}}^{\mathrm{Q},2,3}$$

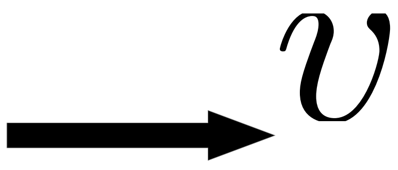


$$\text{Distance}_{\mathrm{P}}^{\mathrm{Q},1,2} = ct_{\mathrm{P}}^{\mathrm{Q},1,2} = L_{\mathrm{P}}^{\mathrm{Q}} + vt_{\mathrm{P}}^{\mathrm{Q},1,2} \quad \text{or} \quad (c-v)t_{\mathrm{P}}^{\mathrm{Q},1,2} = L_{\mathrm{P}}^{\mathrm{Q}}$$
$$\text{Distance}_{\mathrm{P}}^{\mathrm{Q},2,3} = ct_{\mathrm{P}}^{\mathrm{Q},2,3} = L_{\mathrm{P}}^{\mathrm{Q}} - vt_{\mathrm{P}}^{\mathrm{Q},2,3} \quad \text{or} \quad (c+v)t_{\mathrm{P}}^{\mathrm{Q},2,3} = L_{\mathrm{P}}^{\mathrm{Q}}$$
$$\text{Therefore } T_{\mathrm{P}}^{\mathrm{Q}} = t_{\mathrm{P}}^{\mathrm{Q},1,2} + t_{\mathrm{P}}^{\mathrm{Q},2,3} = L_{\mathrm{P}}^{\mathrm{Q}}\left(\frac{1}{c-v} + \frac{1}{c+v}\right)$$
$$= L_{\mathrm{P}}^{\mathrm{Q}}\frac{(c+v)+(c-v)}{c^2-v^2} = \frac{2L_{\mathrm{P}}^{\mathrm{Q}}}{c(1-v^2/c^2)} = 2\gamma^2 L_{\mathrm{P}}^{\mathrm{Q}}/c$$

Consider 'clock' in Q timed from P



$$\text{Time } 1,2 \; = t_{\rm P}^{\rm Q,1,2}$$
$$\text{Time } 2,3 \; = t_{\rm P}^{\rm Q,2,3}$$



$$T_{\rm P}^{\rm Q} = 2\gamma^2 L_{\rm P}^{\rm Q}/c = \gamma T_{\rm P}^{\rm P} = \gamma \frac{2L_{\rm P}^{\rm P}}{c}$$

$$\text{Therefore } \frac{L_{\rm P}^{\rm Q}}{L_{\rm P}^{\rm P}} = \frac{1}{\gamma} \leq 1$$

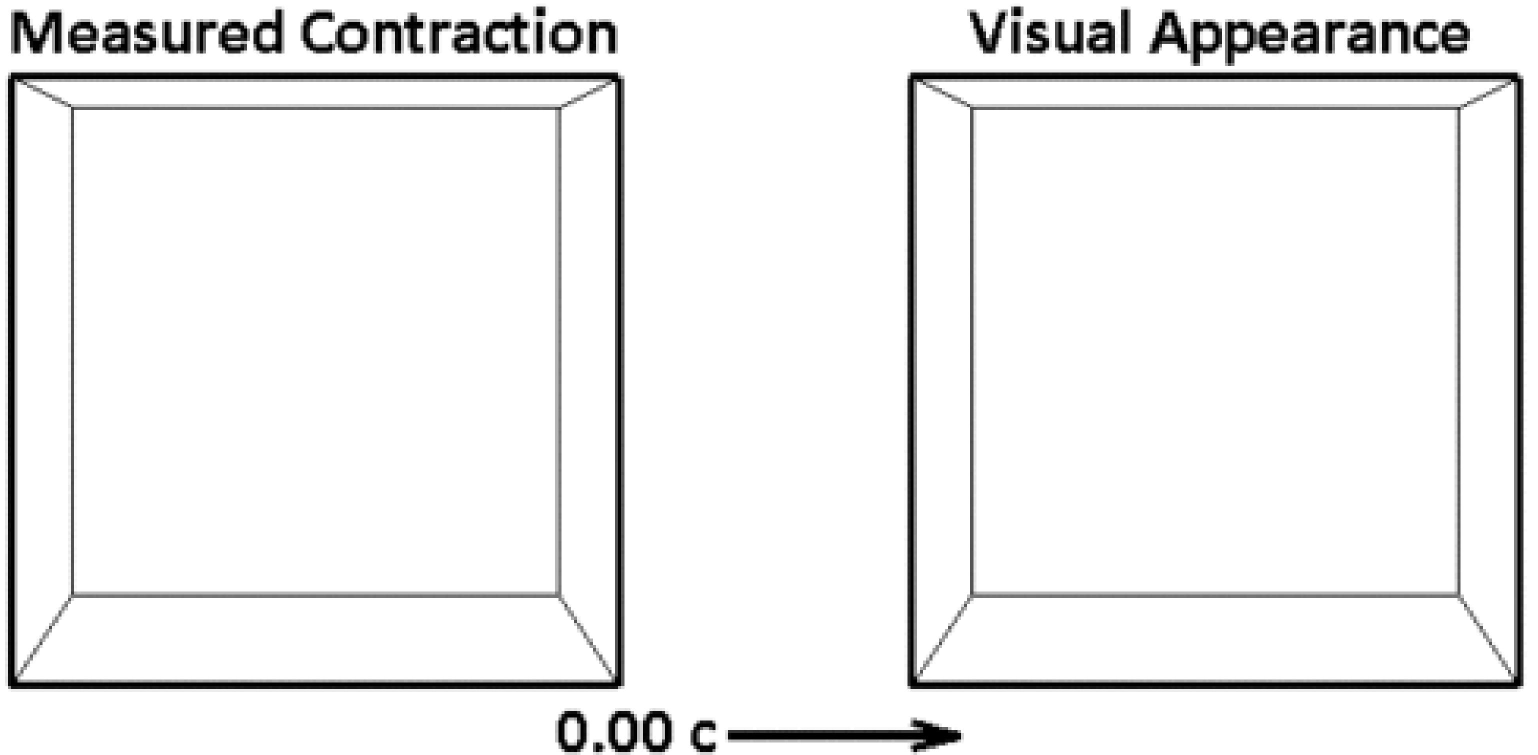
length contraction

Example of length contraction 时间扩张 – muons 渺子

Muons are created in the upper atmosphere 中气层 (e.g. 10 km) from fast, incoming protons 质子 .
They travel close to the speed of light (e.g. 0.98*c*).
They decay with a half-distance corresponding to 600 m.
Few would arrive at the ground.
But because of length contraction ($\gamma \sim 5$)
in *their reference frame* it is only 10/5 = 2 km
to the ground.
So lots get to the ground $(½)^{2/0.6}$=0.10

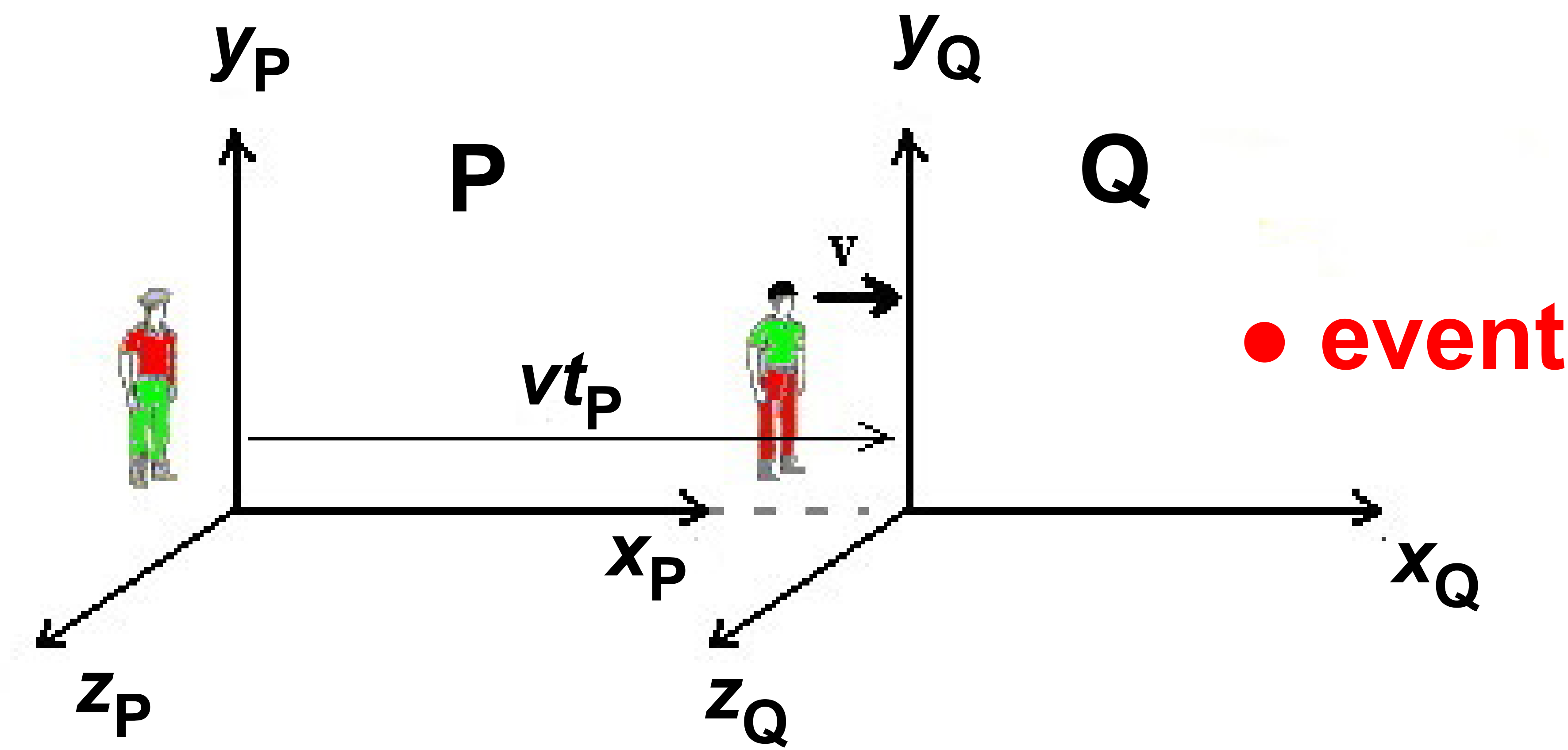
Note :

Light takes time to travel to the observer.
This has not been included in these calculations.
For what we would see – as opposed to infer – must take account of light travel times



Can derive equations relating *events* 事件 in P and Q

An *event* occurs at a particular x, y, z, t



$$x_{\rm Q} = \gamma(x_{\rm P} - vt_{\rm P})$$
$$y_{\rm Q} = y_{\rm P}$$
$$z_{\rm Q} = z_{\rm P}$$
$$t_{\rm Q} = \gamma(t_P - vx_{\rm P}/c^2)$$
$$x_{\rm P} = \gamma(x_{\rm Q} + vt_{\rm Q})$$
$$t_{\rm P} = \gamma(t_{\rm Q} + vx_{\rm Q}/c^2)$$

The Lorentz Transformations 洛伦兹变换

↔

Newtonian :

$x_P = x_Q + vt$

Only one *t*

$= t_P = t_Q$

Simultaneity 同时性 is relative

E.g. Two simultaneous events A, B in Q at time $t_{\rm Q}^{\rm A} = t_{\rm Q}^{\rm B}$

In P they are separated in time by

$$t_{\rm P}^{\rm A} - t_{\rm P}^{\rm B} = \gamma(t_{\rm Q}^{\rm A} + vx_{\rm Q}^{\rm A}/c^2) - \gamma(t_{\rm Q}^{\rm B} + vx_{\rm Q}^{\rm B}/c^2)$$
$$= \gamma v(x_{\rm Q}^{\rm A} - x_{\rm Q}^{\rm B})/c^2$$

Depending on $({\rm x}_{\rm Q}^{\rm A} - x_{\rm Q}^{\rm B})$ A can occur *before* or *after* B in P

But cannot violate cause and effect 因果

$$x_{\rm Q} = \gamma(x_{\rm P} - vt_{\rm P})$$
$$y_{\rm Q} = y_{\rm P}$$
$$z_{\rm Q} = z_{\rm P}$$
$$t_{\rm Q} = \gamma(t_P - vx_{\rm P}/c^2)$$
$$x_{\rm P} = \gamma(x_{\rm Q} + vt_{\rm Q})$$
$$t_{\rm P} = \gamma(t_{\rm Q} + vx_{\rm Q}/c^2)$$

The Lorentz Transformations
洛伦兹变换

Mixing of time and space
→ "space-time" 时空

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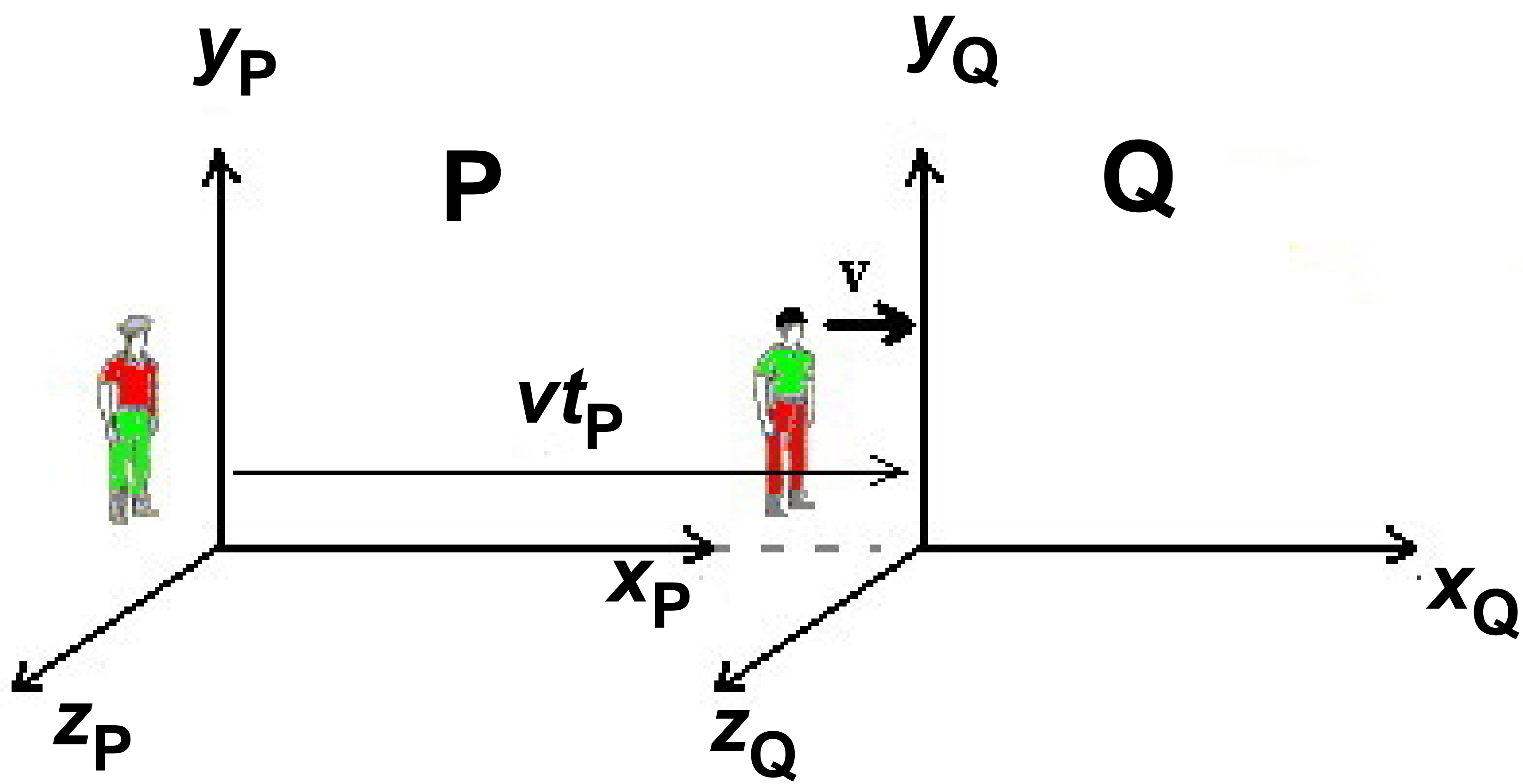
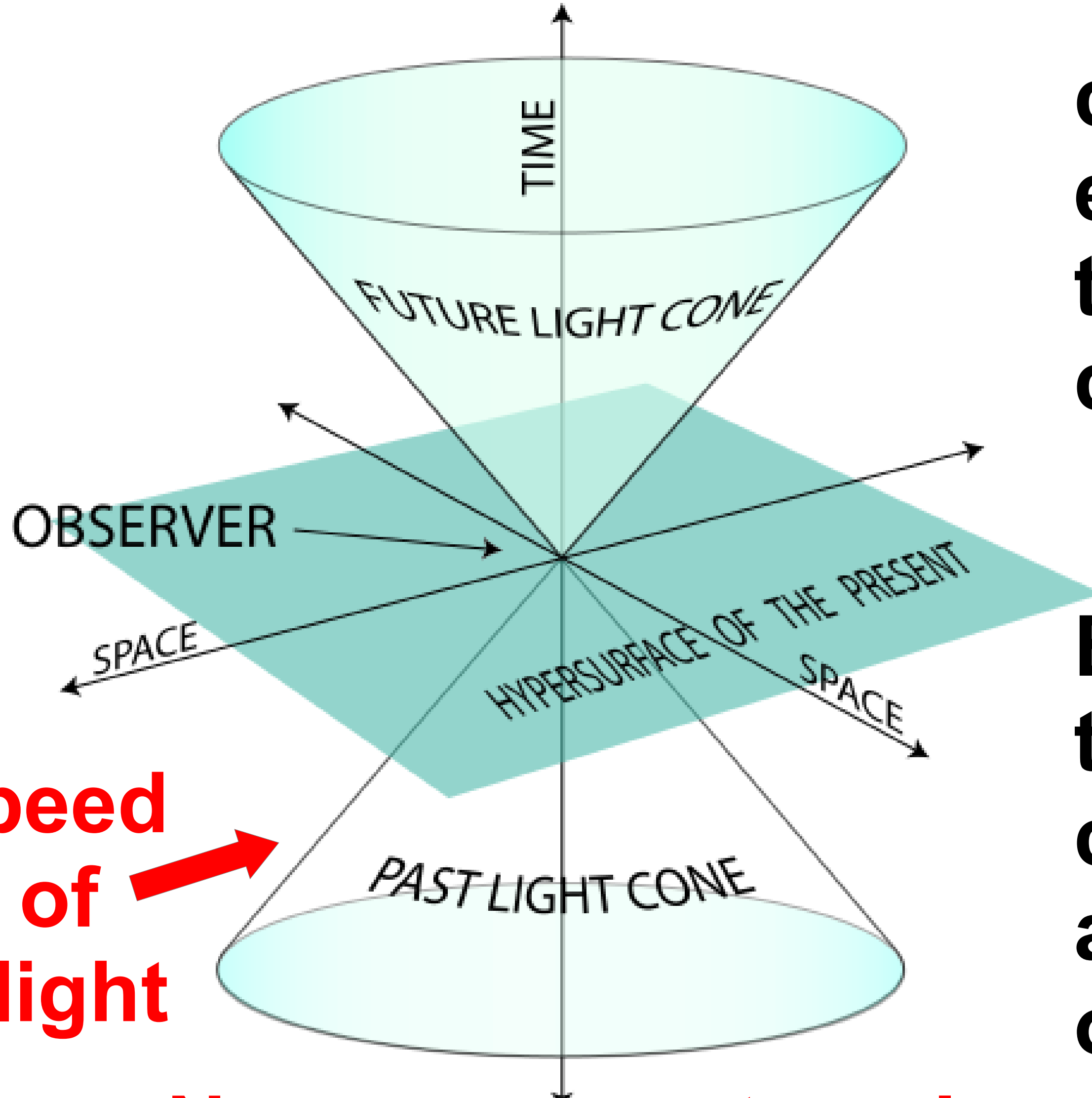


Cause and effect



The observer can affect events inside the future light cone

Events inside the past light cone can have affected the observer

speed of light

No cause can travel faster than the speed of light